

Performance and Mass vs. Operating Temperature for Pulse Tube and Stirling Cryocoolers

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ABSTRACT

Single stage pulse tube and Stirling cryocooler data that were reported in the open literature during the past decade were analyzed to find mathematical scaling laws for estimating the required input power and mass, as functions of the operating temperature at a required capacity, for ambient temperature heat rejection. It is found that the average fraction of Carnot efficiency, determined empirically from an abundance of performance data for each cooler type, follows a relatively smooth nonlinear curve when plotted against the cold head operating temperature. This result can be used to estimate the required input power for a given temperature and capacity requirement. Incorporating results from a previous survey that separately tabulated the cryocooler electronics and thermomechanical masses of many space cryocoolers, the cryocooler total mass can be estimated from the input power requirement using equations derived from the survey results.

INTRODUCTION

The primary reference sources for this study were the International Cryocooler Conference (*Cryocoolers 12 to 15*) and the Cryogenic Engineering Conference (*Advances in Cryogenic Engineering 47, 49, 51, and 53*). In addition, a number of articles were found in journals such as *Physica* and *Cryogenics* that were not replicated in either of the two primary sources. Only single stage Stirling and pulse tube (PT) cryocooler data from tables and figures were analyzed, and care was taken not to include “required performance” or “specifications” that are often presented during cooler development. The experimental rejection temperature, T_h , was used in the analysis, but 300 K was assumed if not stated explicitly. About 95 % of over 120 data sets referenced $T_h = 300$ K, but the rejection temperature ranged from 273 K to 321 K. For a given value of the cold head operating temperature, T_c , lower rejection temperatures typically result in higher experimental efficiencies (COPs), but the fraction of Carnot is less affected, since the Carnot efficiency itself also changes in the same direction as the COP; therefore, no corrections for T_h were made. The range of T_c for the single stage PTs was $35\text{ K} < T_c < 170\text{ K}$ and for Stirling coolers it was $50\text{ K} < T_c < 250\text{ K}$. An attempt has been made to include the most recent data, but there will be some unavoidable omissions and errors.

BACKGROUND

Results from several cryocooler surveys were compared by Kittel in 2007¹. The cryocooler efficiency data in those surveys, typically presented as a % of Carnot efficiency η_{fc} , or fraction of Carnot efficiency η_{fc} , are often plotted against the capacity Q_c (i.e., net cooling power). In some cases the data, although roughly divided into several operating (cold end) temperature ranges and without regard to the cryocooler type, number of stages, or application, are plotted together and the data scatter is extensive. For example, data in the well-known 1974 Strobbridge survey², plotted as η_{fc} vs. Q_c (from 0.1 W to 1 MW), is categorized into low (1.8 K to 9 K), middle (10 K to 30 K), and high (30 K to 90 K) temperature ranges, but without regard to cooler type, and into older and more recent data groups, as shown by the author. It is suggested by Kittel¹ that the Strobbridge survey data indicate that efficiency is primarily a function of capacity Q_c , rather than of operating temperature T_c , although the plotted data do appear to show a systematic temperature dependence — generally higher efficiencies for the higher temperature ranges are obvious at almost all capacities. One best-fit curve to the Strobbridge data is given by the logarithmic expansion (base 10) as

$$\eta_{fc} = 10^{[-1.7356 + 0.599998*(\log Q_c) - 0.1474*(\log Q_c)^2 + 0.021323*(\log Q_c)^3 - 0.00125*(\log Q_c)^4]} \quad (1)$$

where η_{fc} is the fraction of Carnot efficiency and Q_c is the capacity in watts. The ter Brake study³ analyzed 80 K cryocooler efficiency data for the same range of capacity. The 80 K data of ter Brake³, while also scattered, are about a factor of 2.4 above the Strobbridge best-fit curve¹ for capacities less than about 1 kW, indicating a trend toward improved performance at lower capacities. Good agreement between both surveys for these mostly commercial cryocoolers above 1 kW probably reflects the fact that essentially the same scant data are used in the high capacity range. The combined data from these two surveys are reproduced in Figure 1 below¹.

An updated study of space cryocoolers (mostly Stirling and PT) by the AFRL in 2004⁴, for a wide range of temperature, from 6 K to 95 K, but for a much lower range of capacity, 0.2W to ~20W, gives efficiencies that are 2 to 3x higher than the Strobbridge best fit curve⁴, given by the logarithmic expansion (base 10) as

$$\eta_{fc} = 10^{[-1.26281 + 0.45936*(\log Q_c) - 0.08743*(\log Q_c)^2]} \quad (2)$$

where -1.26281 replaces 0.73719 in Strobbridge⁴ to give η_{fc} instead of η_{pc} . The higher efficiency probably reflects the requirement for optimal performance at any T_c for space applications, where cost is not a primary constraint. Kittel suggests that a factor of 3x Strobbridge may be a better correlation for aerospace cryocoolers¹, so that -1.7356 in Equation 1) becomes -1.2587 . (Note that the equation given for the Strobbridge⁴ curve in terms of η_{pc} , has sign errors in the 3rd and 5th terms. For η_{fc} , the lead coefficient 0.264392 in that equation becomes -1.73561 .) An overview of NASA space cryocoolers was given by Boyle and Ross⁵ in 2002, but little performance data were included. A survey by Bruning⁶ for 30 K to 90 K cryocoolers in 1999 (see Fig 5 in Bruning et al.⁶) gives a best-fit simple power law curve as

$$\eta_{fc} = 0.029758 * Q_c^{0.2407} \quad (3)$$

where the coefficient⁶, 2.9758, has been changed here to 0.029758 to give η_{fc} ; again, the operating temperatures do not appear explicitly in this equation. Breedlove⁷ has described very high capacity cryocooler applications and Radebaugh⁸ has discussed recent developments in high capacity PT coolers. He notes that $\eta_{fc} = 0.19$ has been reported at 80 K at a capacity of 300 W⁹. Recently, capacities greater than 1 kW at 80 K were reported by the same group¹⁰ (see Table 1 in Potratz¹⁰) with $\eta_{fc} = 0.17$.

Note that none of the correlations given in Equations 1), 2) or 3) takes into account the observed dependence off η_{fc} on the operational temperature T_c , as will be shown below. In fact, they predict widely different values of η_{fc} for the same value of Q_c . In this study we will postulate that $\eta_{fc} = \eta_{fc}(T_c)$, because experimentally the cryocooler coefficient of performance, or COP, defined as the ratio of capacity to input power, Q_c/P_{in} , is found to be a monotonic increasing function of T_c

with a temperature dependence different from the Carnot efficiency $\eta_c = T_c/(T_h - T_c)$, for a fixed heat rejection temperature T_h . It is noteworthy, however, that at least one analysis at AFRL has recognized the complex dependence of input power on not only Q_c but also T_c and T_h , and consequently η_{fc} ¹¹. The parameters for that model, developed specifically for the NGST 150 K mini PT cryocooler, are given in Table 1 in Abhyankar¹¹.

Previous surveys have shown that correlations between cryocooler mass and capacity, or mass and DC input power, appear to obey simple power laws (and are commonly modeled as such) when a wide range of data at various operating temperatures and for various cryocooler types are considered. The physical basis for the apparent correlations, independent of cooler type, is that regenerative cryocoolers, such as PTs and Stirlings, require larger more massive compressors (pressure oscillators) to achieve higher capacities, while recuperative cryocoolers, such as Turbo Braytons, require more massive heat exchangers^{1,7}. The survey by Bruning⁶ for 30 K to 90 K coolers gives (see Fig 3 in Bruning et al.⁶)

$$M = 7.62 * Q_c^{0.72} \quad (4)$$

where M is (presumably) the total cooler mass in kg and Q_c is the capacity in watts. This general result does not account for the dependence of M on T_c at fixed maximum input power, which might require a temperature-dependent coefficient in Equation 4). A direct correlation in terms of the DC input power is given in Kittel¹ for ter Brake's data and for single stage cooler data⁴, with operational temperatures greater than 65 K, as

$$M = 0.0711 * P_{in}^{0.905} \quad (5)$$

where M is (presumably) the total cooler mass in kg and P_{in} is the DC input power in watts. A similar correlation for the space cryocoolers is given⁴ with a coefficient of 0.082 and an exponent of 0.998. In Kittel¹ the coefficient in Equation 5) is changed to 0.1422 for the analysis of coolers operating at 20 K, which in effect implies that the COP must be a temperature-dependent variable, with more power and therefore greater mass required to achieve a given capacity at a lower operating temperature. A chart given by Radebaugh¹² for cryocoolers with an input power range of ~6W to 40kW includes data from two very high capacity regenerative cryocoolers operating at 77 K – the Stirling Corporation Model SPC-4 at 4 kW with $\eta_{fc} = 0.26$ and the Praxair PT Model HF at 1.1 kW with $\eta_{fc} = 0.14$. These data points fall near the line given by Equation 5).

If Q_c in Equation 4) is expressed in terms of P_{in} and T_c through $\eta_{fc}(T_c)$, determined empirically from the extensive PT and Stirling cryocooler published data, a better estimate for the cryocooler mass might be obtained for a given capacity and cold end temperature application. However, Q_c can be associated with a continuous range of input powers above a threshold value, so to determine the minimum cooler mass required we need to establish a correlation between the cooler mass and its maximum input power. (A caveat here is that maximum compressor efficiency is sometimes not obtainable at maximum input power and so a de-rating factor must be included in the mass estimate in these cases.) It is further suggested that although the maximum cooler mass appears to obey a simple power law, as in Equations (4) or (5), such correlations are inadequate to model the cooler mass over the widest range of capacity or input power.

SURVEY RESULTS AND ANALYSIS

The current study shows that the average efficiency obtained from many data sets depends strongly on the cold end temperature T_c . This comparative analysis focuses specifically on Stirling and Stirling-type PT cryocoolers and analyzes the most recent data available (1999 to 2009) in terms of efficiency at specific operating temperatures. Thus an average efficiency at any given temperature can be used to calculate the average DC input power for a required capacity. Furthermore, for the "best data," i.e., data falling above the average efficiency, where the limits of the state-of-the-art designs may be realized, a best-case lowest input power and mass can be determined.

In estimating the total cryocooler system mass, it is desirable to consider the cryocooler electronics (CCE) mass as well as the thermo-mechanical unit (TMU) mass. An analysis of the PT and

Stirling data given in an Aerospace Corporation Report¹³ in 2000 for space cryocoolers shows that for high cooling powers, and therefore high total mass, the fraction of total mass allocated to the thermo-mechanical unit (TMU mass), i.e., the pressure generator and cold head, trends to about 0.86, while the CCE mass fraction trends to about 0.14, as the total mass increases. This report contains the most comprehensive mass data that could be found in this survey. Data from a more recent report by Aerospace Corporation is consistent with this result¹⁴.

Figure 1 plots the fraction of Carnot η_{fc} against the capacity Q_c for Stirling and PT coolers at several operating temperatures from this survey. Although the data scatter is not unlike that of Figure 2, categorizing the data in terms of operating temperature partially resolves this issue. The power law curves where given should be taken as “guides to the eye” to show that η_{fc} is an increasing function of temperature at a given capacity for each cooler type over this limited temperature range, 50 K < T < 80 K. The two heavy horizontal dashed lines, however, show that the best values

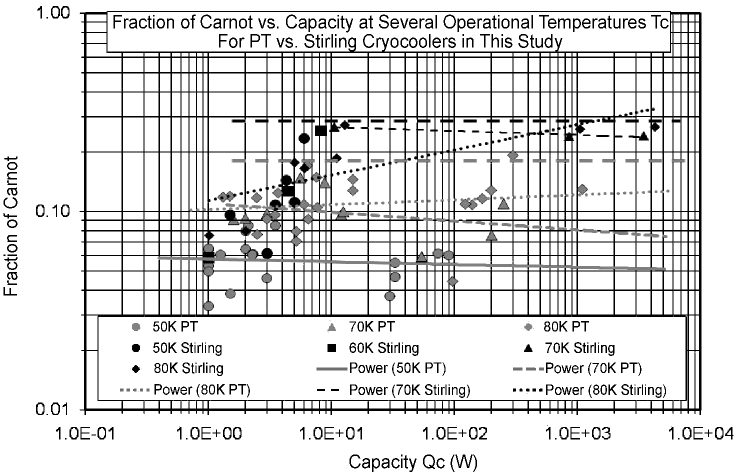


Figure 1. The Fraction of Carnot η_{fc} vs. Capacity Q_c and Temperature T_c for PT and Stirling cryocooler data in this survey. PT data and lines are gray, Stirling are black. Heavy dashed lines are best data at 80 K.

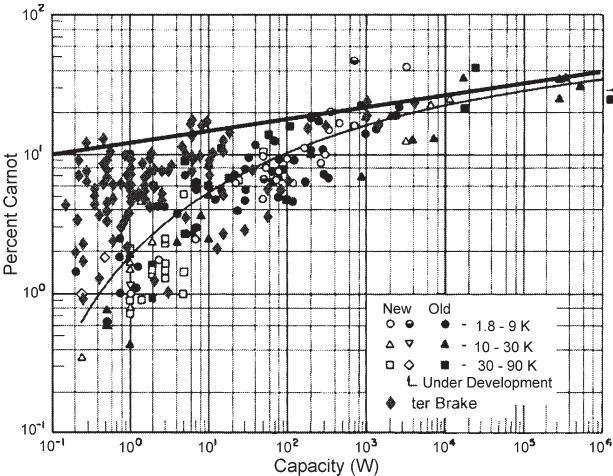


Figure 2. The 2002 ter Brake and 1974 Strobridge surveys combined, as given in Fig. 2 of Kittel¹. In the legend “New” is circa 1974 and “Old” is prior to 1974. Note that the ter Brake limit (upper solid line) delineates a boundary of maximum efficiency.

of η_{fc} (at 70 K and 80 K) are essentially independent of Q_c over a capacity range of $\sim 10\text{W}$ to 4000W (upper line, Stirling) and $\sim 5\text{W}$ to 1100W (lower line, PT). These data indicate a continuing trend toward more efficient low capacity coolers, effectively pushing the lower end of the ter Brake limit in Fig. 1 upward and flattening its slope for $Q_c > 10\text{W}$.

Figure 3 shows the results of the current survey (1999 to 2009 data only) for the average fraction of Carnot efficiency, η_{fc} , and the average coefficient of performance $\langle \text{COP} \rangle$ vs. the operating temperature T_c for many single stage Stirling and Stirling-type PT cryocoolers at many temperatures from 35 K to 250 K. Despite the $[1/(n-1)]^{0.5}$ weighting factor, the standard deviations for η_{fc} are indicative of the variability of data samples at each temperature. Figure 3 includes about 120 data points from the current survey. $T_h = 300\text{ K}$ is the predominant heat rejection temperature, but the variation of T_h is from 273 K to 321 K for some data. In all calculations the actual rejection temperatures were used to compute η_{fc} . If not explicitly stated by the author, T_h was assumed to be 300 K. To model the $\langle \text{COP} \rangle$ and η_{fc} data smoothly the lowest order polynomial was chosen. The best fit quadratic is sufficient for the $\langle \text{COP} \rangle$ and the best fit 3rd order polynomial suffices for η_{fc} — unfortunately, there is scant data in the range from 100 K to 250 K. The $\langle \text{COP} \rangle$ and η_{fc} curves are forced to pass through the origin, since the COP is observed to approach zero faster than the ideal Carnot efficiency as $T_c \rightarrow 0$. That η_{fc} falls off above a maximum value can be attributed to the fact that the Carnot efficiency is increasing rapidly toward infinity as $T_c \rightarrow T_h$, while the $\langle \text{COP} \rangle$ is increasing much more slowly with T_c , as shown by the data in Figure 3. Note that by definition the $\langle \text{COP} \rangle$ and η_{fc} have identical values at 150 K for $T_h = 300\text{ K}$. Similar curves have been shown in the literature (although rarely) in combination with cooling capacity¹⁵, but these can be easily generated from individual cryocooler performance load curves over more limited temperature ranges^{16,17}. An important feature regarding the η_{fc} curve is that it is necessarily a double-valued function, so that plots of η_{fc} vs. capacity (Figure 1) or input power without specifying the operating temperatures are confusing at best. Ideally the data of Figures 1 and 2, averaged by temperature at each capacity, would generate a family of similar η_{fc} curves.

Figures 4 and 5 break out the combined data from Figure 3 into PT and Stirling cooler types, respectively, and also identify the best efficiency data for each type. In Figure 4 the best-fit curve for the pulse tube coolers is given by

$$\eta_{fc} = 1.0139\text{E-}08 * T_c^3 - 9.3879\text{E-}06 * T_c^2 + 1.9057\text{E-}03 * T_c \tag{6}$$

It has a maximum value of ~ 0.11 near 130 K. For $T < 100\text{ K}$, the best data, shown as circles, lie about a factor of two above the curve with $\eta_{fc} = 0.21$ at 77 K¹⁸ and $\eta_{fc} = 0.19$ at 80 K⁹. The dashed

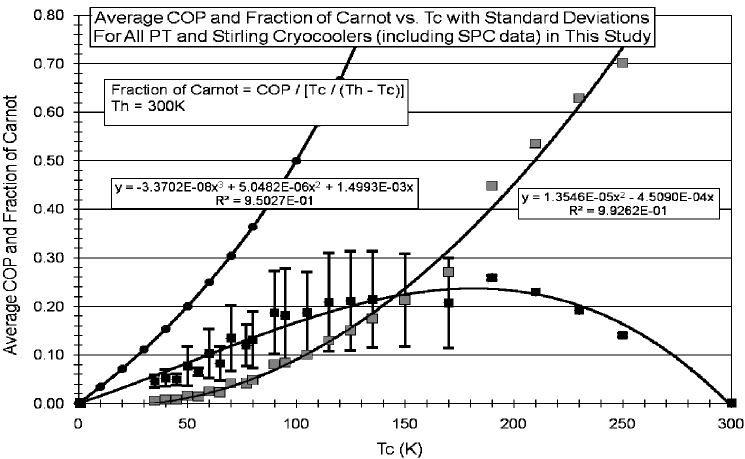


Figure 3. Average COP and Average Fraction of Carnot Efficiency vs. T_c for combined Stirling and PT Data. Smaller black squares are η_{fc} ; larger gray squares are $\langle \text{COP} \rangle$. Black circles (upper line) show the theoretical maximum (Carnot) efficiency.

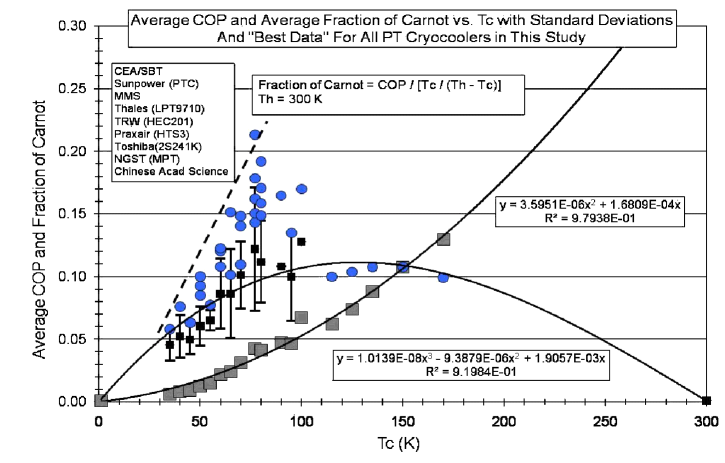


Figure 4. Average COP and Fraction of Carnot Efficiency vs. Tc for PT Data. Black squares are η_{fc} ; gray squares are $\langle \text{COP} \rangle$. Best hfc data at each temperature are the circles near or above the best-fit curve.

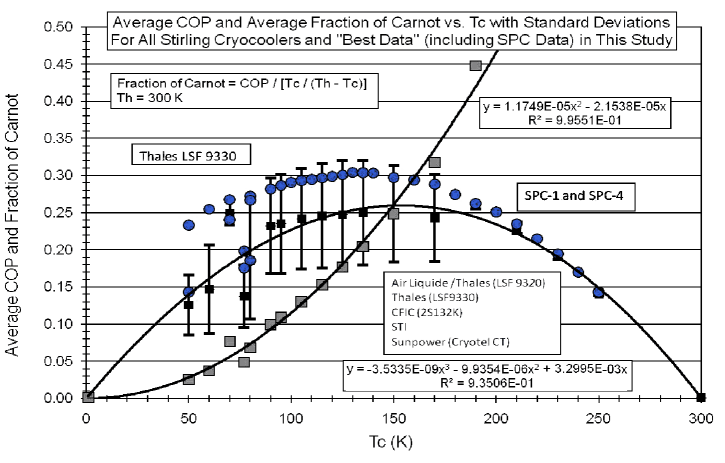


Figure 5. Average COP and Fraction of Carnot Efficiency vs. Tc for Stirling Data. Black squares are η_{fc} ; gray squares are $\langle \text{COP} \rangle$. Best hfc data at each temperature are the circles near or above the best-fit curve. The uppermost circles from 50 K to 80 K are from the Thales model LSF 9330, and the uppermost circles from 80 K to 250 K are from the Stirling Corporation models SPC-1 and SPC-4.

line indicates the current performance “boundary” in terms of η_{fc} and indicates its strong dependence on operating temperature. These above-average efficiencies imply that similar results might be obtainable at $T > 80$ K, but they would have to be consistent with the qualitative nature of the η_{fc} best-fit curve and exhibit a maximum at some higher temperature. For the PT $\langle \text{COP} \rangle$ the best-fit curve is given by

$$\text{COP} = 3.5951\text{E-}06 * \text{Tc}^2 + 1.6809\text{E-}04 * \text{Tc} \tag{7}$$

At 80 K the COP is ~ 0.04 but the best data give ~ 0.08 . In Equations 6) and 7), η_{fc} and COP have values of ~ 0.11 at 150 K.

Figure 5 shows comparable data for Stirling coolers. Note that here η_{fc} and COP are ~ 0.26 at 150 K. The best-fit curves are given by

$$\eta_{fc} = -3.5335E-09 * T_c^3 - 9.9354E-06 * T_c^2 + 3.2995E-03 * T_c \tag{8}$$

and

$$COP = 1.1749E-05 * T_c^2 - 2.1538E-05 * T_c \tag{9}$$

As expected the curves given by Equations 6) and 7) for PTs fall below the PT/Stirling combined curves shown in Figure 3, while the curves given by Equations 8) and 9) for Stirlings are well above those curves (note that the ordinate scales differ in Figures 3 to 5). While the qualitative nature of the Stirling η_{fc} curve is similar to the PT η_{fc} curve, the peak value is shifted to the right toward ~ 150 K. This shift results from the highly efficient Stirling Corporation SPC data for $T_c > 170$ K. In Equation 8) $\langle \eta_{fc} \rangle$ has a maximum value of ~ 0.26 near 150 K. For $T_c < 200$ K, the best data, shown as circles, lie consistently above the curve with $\eta_{fc} = 0.27$ at 80 K for the Thales LSF 9330¹⁹ and $\eta_{fc} = 0.30$ at 130 K for the Stirling Corporation SPC-4²⁰. Taken together these two data sets constitute the current state-of-the art performance for single stage Stirling coolers over the range $50 < T_c < 250$ K.

Although the Stirling data average efficiencies are typically from $\sim 100\%$ ($50 < T_c < 100$ K) to as much as $\sim 160\%$ ($100 < T_c < 170$ K) greater than their PT counterparts, it should be noted that the efficiency gap for “best data” has been narrowed considerably in recent years. At 70 K the Stirling efficiency advantage is only 80 %. A further example at 77 K is the recent data of Hu, reducing the gap to $\sim 30\%$ ¹⁸. As noted in the Introduction, the advantage of using the η_{fc} , as opposed to the COP, as a measure of cooler performance efficiency is that for values of T_h reasonably close to the T_h reference value (here 300 K), η_{fc} is less affected by the T_h variability than is the COP. However, the rejection temperature given¹⁸ is $T_h = 283$ K. This low value of T_h may partly account for the high PT efficiency; results at $T_h = 300$ K would be more useful for a direct comparison to the best Stirling coolers.

Figures 6 and 7 show the qualitative dependence of the cryocooler TMU mass on the operating temperature as a function of the capacity. Figure 6 shows mass data categorized by temperature T_c ¹³, and Figure 7 shows similarly categorized TMU mass data obtained in the current study. In Figure 6 both regenerative and recuperative cooler TMU masses are plotted along with a single calculated TMU mass value at 1100 W. Although the curves in Figure 6 would cross at $Q_c \sim 10^4$ W (for lack of sufficient high capacity data), the general result is that there is a strong dependence of the cryocooler mass on the operating temperature at any given capacity, as indicated by the hierarchy of the best fit power law curves. Those equations are ordered from lowest to highest temperature categories at the right side in each figure. In Figure 7 the power law slopes (~ 0.78) are similar to that of Equation 4), derived from earlier data covering a larger temperature

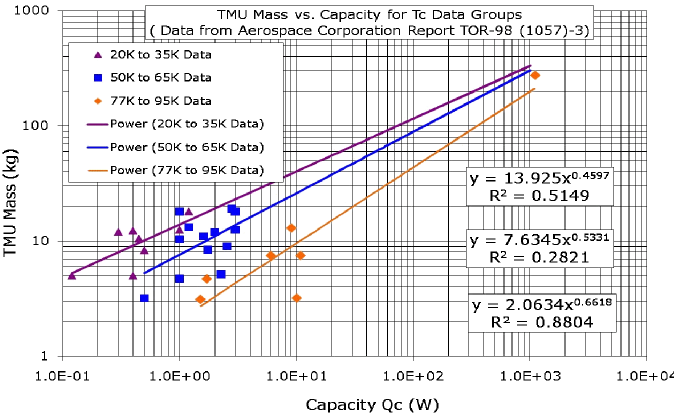


Figure 6. TMU Mass vs. Capacity from the Aerospace Corporation Survey data¹³. The single point at $Q_c = 1100$ W is from Potratz¹⁰, calculated using the compressor mass from CFIC.

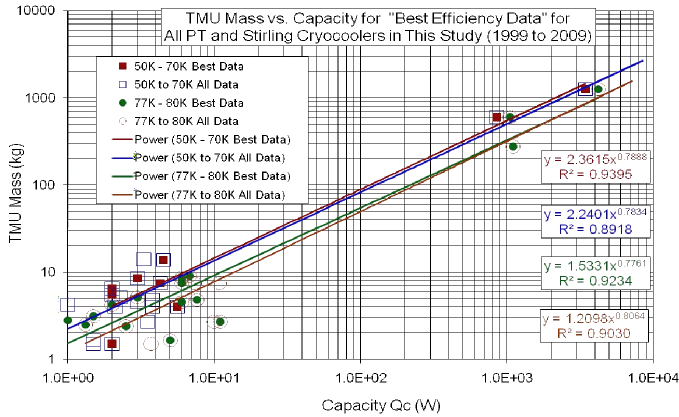


Figure 7. TMU Mass vs. Capacity for the Current Study. “All data” and “best data” results are essentially the same curves for each range of T_c – power law coefficients are indicative of the mass and are about 1.6x greater for the lower T_c range. Uppermost points from Potraz¹⁰ and the Stirling Cryogenics website²⁰, are calculated using the compressor mass from CFIC²¹.

range, $30\text{ K} < T_c < 90\text{ K}$, but the coefficients vary inversely with T_c . Therefore, Equation 4) is qualitatively consistent with the Figure 7 best-fit power law curves obtained from the more recent data, but it predicts a larger TMU mass by about a factor of four. It could be generalized by using a temperature-dependent coefficient, e.g., $C = 1100 / T_c^{1.5}$, to give

$$M = 1100 * Q_c^{0.78} / T_c^{1.5} \quad (10)$$

Figure 8 shows a plot of the cryocooler mass data (given in kg)¹³ and five additional points from this survey. Here the TMU mass and the CCE mass are plotted as fractions of the total cryocooler mass M ($=$ TMU mass + CCE mass) to determine simple linear asymptotic relationships that allow an estimate of the total mass when only one mass component is given. These linear best fits, valid for total mass $M > 10\text{ kg}$ ²², are given by

$$\text{TMU Mass} = 0.8558 * M - 2.9876 \quad (11)$$

and

$$\text{CCE Mass} = 0.1442 * M + 2.9876 \quad (12)$$

from which either the TMU mass or the CCE mass can be calculated when only one of those two masses is provided. Typically, only the TMU mass is given, so that

$$\text{CCE Mass} = 0.168 * \text{TMU Mass} + 3.49, \quad \text{TMU Mass} > 5\text{ kg}. \quad (13)$$

Figure 9 shows the mass vs. input power data given in the Aerospace Corporation Report¹³. These are the same TMU mass data from Figure 6, the total mass data are also included here and both are now plotted against the (maximum) input powers given in that report. The simple power law curve fits are given by

$$\text{TMU Mass} = 0.2790 * P_{in}^{0.7207} \quad (14)$$

$$M = 0.9438 * P_{in}^{0.5769} \quad (15)$$

In the current survey only 5 of the 46 references that give the TMU mass data also include the CCE mass, and since those 5 data points were consistent with Equations 11) to 13), those equations were used to estimate the CCE and total masses for the other 41 points. The resulting simple power law curve fits (not shown) are given by

$$\text{TMU Mass} = 0.1597 * P_{in}^{0.7133} \quad (16)$$

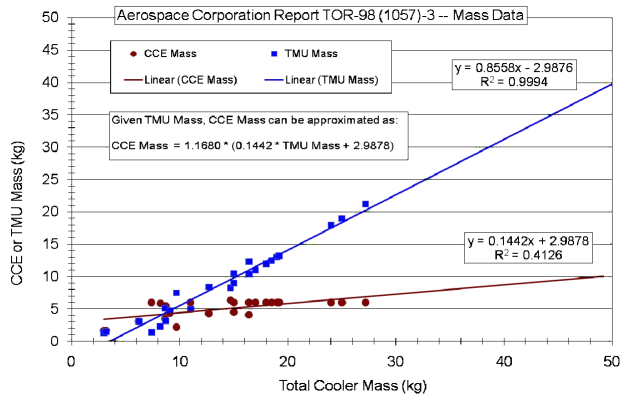


Figure 8. TMU Mass and CCE Mass vs. Total Cryocooler Mass (kg) from Donabedian¹³.

$$M = 0.6702 * P_{in}^{0.5588} \tag{17}$$

These results, based on the more recent data, predict almost identical input power dependences for the TMU and total masses as Equations 14) and 15)¹³ data, but they suggest that the TMU and total masses are lower by ~57% and ~71%, respectively. Because the P_{in} power law dependence of Equation 5) differs significantly from that of Equation 17), Equation 5) predicts a lower total mass for $P_{in} < 652 \text{ W}$ but a higher total mass for $P_{in} > 652 \text{ W}$. Therefore, Equation 17) indicates that large scale coolers might be optimized for lower masses than have previously been achieved. Also note that Equations 14) and 16) correctly predict a lower total mass than Equations 14) and 16) for the TMU mass alone at input powers above ~5 kW, because the relatively smaller CCE mass contribution is properly weighted in those curve fits.

Figure 10 plots TMU mass and total cryocooler mass vs. DC input power based on compressor mass data that have been published on the CFIC website²¹. To obtain the approximate TMU mass, the compressor (pressure oscillator) mass has been scaled up by a factor of 1.2 to include the estimated TMU cold head mass (~20%). Using Equation 13) the CCE mass has been calculated and the total cryocooler mass then computed. Lacking hard data for the masses of the several large-scale cryocoolers that have been reported in the open literature, the CFIC database perhaps provides a

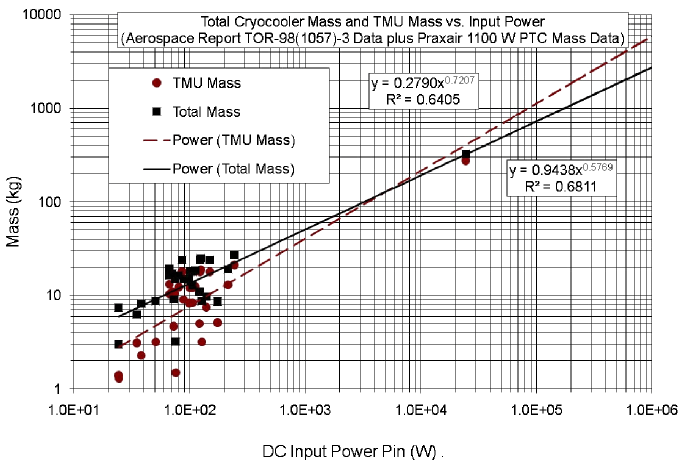


Figure 9. Simple Power Law Result for Total Cryocooler Mass and TMU Mass vs. DC Input Power for data from Donabedian¹³.

reasonable and consistent source for estimating the power-mass relationship for large PT or Stirling cryocoolers. The lowest two points on the curves were computed using Equations 15) and 16) and also to specify the limiting intercepts at $P_{in} = 1$ W on the log-log graph. It can be seen in Figure 10 that there is a nonlinear dependence of the mass on the input power at both low and high powers with an inflection point near $\log(P_{in}) = 3.5$, or about 3.16 kW. This result shows that a simple power law (as shown by the dashed line) does not accurately describe the mass vs. input power relation over a broad range of input power when a consistent set of data is used; there is clearly an economy of scale with respect to the cooler mass as the power is increased. For example, the CFIC 10 kW compressor weighs 160 kg, while the 20 kW compressor weighs only 230 kg — only 44 % more compressor mass for a 100% increase in power. Note that the two Stirling Corporation SPC models have larger masses than the CFIC estimates, but the rate of change of mass with respect to input power is consistent with the CFIC curve fit. The polynomial curve fit in Figure 10 gives the PT or Stirling cryocooler total mass M (kg) in terms of the DC input power (W) as a log (base 10) expansion:

$$M = 10^{\{-5.5260E-02 * [\log(P_{in})]^3 + 5.5651E-01 * [\log(P_{in})]^2 - 1.0931E+00 * \log(P_{in}) + 1.3315E+00\}} \tag{18}$$

APPLICATION OF SURVEY RESULTS TO A 5 KW CAPACITY REQUIREMENT

The results of this survey can be applied to estimate the required mass of a large regenerative cryocooler having a required capacity Q_c and operating at a required temperature T_c with a rejection temperature $T_h = 300$ K. Using the result for the average fraction of Carnot efficiency for Stirling cryocoolers, as shown in Figure 5 and modeled in Equation 8), an operating temperature of 50 K gives $\eta_{fc} = 0.14$. From the definition by which η_{fc} is computed,

$$P_{in} = (Q_c / f_d * \eta_{fc}) * (T_h - T_c) / T_c \tag{19}$$

where a *de-rating factor* f_d has been included to account for a possible decrease in compressor efficiency at maximum input power. If we require $Q_c = 5000$ W and assume $f_d = 1$, then $P_{in} = 1.786E+5$ W. By Equation 17), the total mass $M = 862$ kg. A better result would use $\eta_{fc} = 0.23$, as given by the best data in Figure 5, for which Equation 18) then gives $P_{in} = 1.087E+5$ W and Equation 17) gives $M = 763$ kg. These results would be slightly less favorable if, for example, we set $f_d = 0.90$, thereby requiring $P_{in} = 1.984E+5$ W for $\eta_{fc} = 0.14$, giving $M = 877$ kg, and $P_{in} = 1.208E+5$ W for

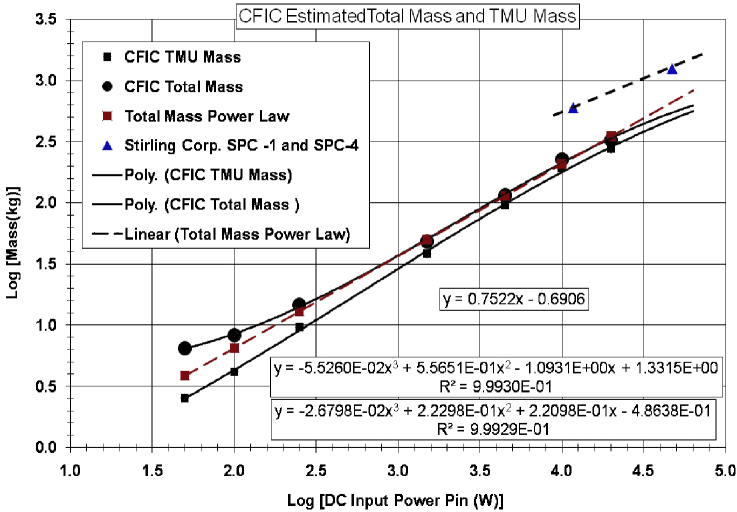


Figure 10. Total Cryocooler Mass and TMU Mass vs. DC Input Power plotted on log (base 10) scales. The range of validity for these approximations is ~ 50 W < P_{in} < $\sim 2E+5$ W.

$\eta_{fc} = 0.23$, giving $M = 787$ kg. The simple power law fit shown in Figure 10 gives 1819 kg and 1252 kg for the $f_d = 1$ input powers and 1819 kg and 1506 kg for the $f_d = 0.9$ input powers.

The results of this example can be compared to the predictions of the various formulas from previous surveys listed in the Background section above. Using $Q_c = 5000$ W in Equations 1), 2), and 3) gives $\eta_{fc} = 0.205$, 0.174, and 0.231, respectively, independent of the operating temperature T_c . Using $Q_c = 5000$ W in Equation 4a) gives $M = 3509$ kg, or better, Equation 4b) gives $M = 2388$ kg. Similarly, assuming $\eta_{fc} = 0.23$ and using $P_{in} = 1.087E+5$ W in Equation 5) gives $M = 2568$ kg. These results for the estimated total mass are somewhat higher than those found using the simple power law of Figure 10 and quite high compared to 862 kg ($\eta_{fc} = 0.14$), or 763 kg ($\eta_{fc} = 0.23$), determined from the polynomial curve fit (Equation 18), where P_{in} is computed using actual operating temperature data for η_{fc} and where the economy-of-scale correlation for mass vs. P_{in} using CFIC data is considered in the overall analysis. Similar but less favorable results can be obtained for PT coolers using $\eta_{fc} = 0.06$ or $\eta_{fc} = 0.10$ (best data) from Figure 4.

The results presented here will require frequent updates given the rapid experimental progress in cooler performance, especially with regard to high capacity coolers. PT cooler efficiencies may in time equal Stirling efficiencies, but for now the current status is shown in Figures 4 and 5. The choice of cooler type for a given application depends on many other factors besides best efficiency. However, it would be useful to perform similar surveys for other cooler types based on their operating temperatures and to verify the mass economy of scale vs. input power indicated by Equation 18).

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